

Two-oscillator model of trapped-modes interaction in a nonlinear bilayer fish-scale metamaterial

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Abstract. We discuss the similarity between the nature of resonant oscillations in two nonlinear systems, namely, a chain of coupled Duffing oscillators and a bilayer fish-scale metamaterial. In such systems two different resonant states arise which differ in their spectral lines. The spectral line of the first resonant state has a Lorentzian form, while the second one has a Fano form. This difference leads to a specific nonlinear response of the systems which manifests itself in appearance of closed loops in spectral lines and bending and overlapping of resonant curves. Conditions of achieving bistability and multistability are found out.

Keywords: coupled oscillators, Duffing oscillator, metamaterial, trapped mode, bistability.

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1. Introduction

The resonant phenomena are inherent to all types of vibrations or waves and a number of resonant states in mechanic, acoustic, electromagnetic and quantum systems are well known. Importantly, there are some standard approaches for describing such different resonant phenomena in various branches of physics which are primarily developed within the oscillation theory framework. Within this theory, despite the differences in the nature of the resonant phenomena, they are described from a unified standpoint with similar or even the same equations and techniques.

A resonance is thought to be an enhancement of the response of a system to an external excitation at a particular frequency. It is referred to the resonant frequency or natural frequency of the system. From the oscillation theory standpoint a resonance is introduced by the means of a harmonic oscillator being under an action of a periodic driving force. When the frequency of the driving force is close to the eigenfrequency of the oscillator, the amplitude of oscillations grows toward its maximal value. Besides that, many physical systems may also exhibit the opposite phenomenon when their response is suppressed under certain resonant conditions which sometimes is named with the term *antiresonance* [1]. The simplest example can be illustrated using two coupled harmonic oscillators, where one of them is driven by a periodic force. Remarkably such a system of two coupled harmonic oscillators simultaneously supporting resonant-antiresonant states is considered as an intuitive and popular model to describe features of many resonant phenomena, including electromagnetically induced transparency (EIT) [2, 3], stimulated resonant Raman effect [4], level repulsion [5], conditions for adiabatic and diabatic transitions [6, 7], quantum coherence-decoherence [8], etc.

Thereby in the system of two coupled oscillators, in general, there are two resonances located close to the certain eigenfrequencies of each oscillator [9]. One of the resonances of the forced oscillator demonstrates the standard amplitude growing near its eigenfrequency and it has a *symmetric* spectral line, described by Lorentzian function. At the same time the other resonance demonstrates an unusual sharp peak in the amplitude and it has an *asymmetric* spectral line, known as Fano profile. At the antiresonant state there is a total suppression of the amplitude of the forced oscillator at the eigenfrequency of the second oscillator which is one of its basic properties, originated from the resonant destructive interference, that distinguishes the Fano resonance among the other ones [9].

The resonant-antiresonant states were originally studied in quantum physics in relation to asymmetrically shaped ionization spectral lines of atoms and molecules. But in the recent years they attract appreciable attention in the field of plasmonic nanoparticles, photonic crystals, and then electromagnetic metamaterials [1, 10, 11, 12, 13]. This interest is stimulated by promising applications of resonances with asymmetric spectral lines in sensors, lasing, switching, nonlinear and slow light devices, due to the steep dispersion of their profile. Despite the fact that the nature of such resonances in photonic devices is quite complicated and it is explained by the interference effect

between a certain non-radiative mode and a continuum of radiative electromagnetic waves, the simple two-oscillator model is still widely used to reveal the main resonant features of the optical systems [14, 16, 17].

Another important characteristic of structures supporting resonant-antiresonant states is their possibility to provide an enhanced energy storing. Remarkably, simultaneous presence of both steep resonant feature and strong field localization brings a possibility of realization an optimal bistable switching in nonlinear systems. In particular, in optical systems, the main idea of using the resonant-antiresonant states for all-optical switching and bistability is to introduce an element with nonlinear characteristic and achieve a stepwise nonlinearly-induced shift of the resonant frequency [18, 19, 20, 21, 22, 23, 24]. Thus, by employing such nonlinear shift one can reach bistability in many devices suggested on the plasmonic, photonic crystal and metamaterial platforms.

From the viewpoint of the oscillation theory, a study of resonant-antiresonant states in nonlinear systems causes derivation of the particular model of a chain of two *nonlinearly* coupled oscillators [25]. In a mathematical form such a system can be described by a set of two coupled Duffing equations which is the basic model for illustrating synchronization phenomenon and related effects [26, 27, 28]. It is known that, despite the apparent simplicity of the Duffing equations, there are no easy ways to find an exact analytical solution for the corresponding system of nonlinear equations. In this regard, for its solution some asymptotic approaches are traditionally used. Howbeit obtained solution can comprise a number of peculiarities and stand out by the presence of hysteresis, several stable cycles, complex dynamics and chaotic regimes.

The complete study of the system of nonlinear equations supposes involving a concept of dynamical systems [29]. However, in this paper we intend to restrict ourself only to extension the results of [9] by adding a weak nonlinearity to the system of two coupled oscillators and solving it with the slowly varying amplitude approximation in the frequency domain. The main purpose of the paper is to reveal general changes in the spectral line of the resonant-antiresonant states when such weak nonlinearity is introduced into the system.

Then on the basis of the results obtained from the nonlinear two-oscillator model we propose an example of a nonlinear metamaterial configuration which operating regimes qualitatively resemble characteristics of the mentioned oscillating system. As such a structure we consider a special class of metamaterials which involves planar metasurfaces supporting so-called “trapped-modes”. The trapped-mode is a specific resonant state that appears in the metamaterials made of subwavelength metallic or dielectric particles (inclusions) with a certain asymmetric form [11, 12, 30, 31]. The trapped-modes are the result of *antiphase* oscillations of fields on the particles parts (arcs) and are excited by an external electromagnetic field. In literature such trapped-mode metamaterials sometimes are also referred to EIT-like metamaterials [32]. It is due to the fact that their response is a direct classical analog of EIT because the weak coupling of the antiphased local fields to free space is reminiscent of the weak probability for photon

absorption in EIT observed in atomic system. In this paper we consider a particular configuration of such metamaterial, namely, a nonlinear bilayer fish-scale structure [33].

2. Two-oscillator model: Set of coupled Duffing equations

Our objective here is to study the main spectral features of a chain of two coupled nonlinear oscillators. For this reason we consider the two harmonic oscillator system which models classically the Fano resonance [9]. If it is supplemented by the cubic nonlinear terms, we arrive to the set of two coupled Duffing equations related to the coordinates x_1 and x_2 in the form

$$\begin{aligned} \frac{d^2 x_1}{dt^2} + 2\delta_1 \frac{dx_1}{dt} + \omega_1^2 x_1 + \omega_1^2 \beta_1 x_1^3 - c_1 x_2 &= P_0 \cos(\omega t), \\ \frac{d^2 x_2}{dt^2} + 2\delta_2 \frac{dx_2}{dt} + \omega_2^2 x_2 + \omega_2^2 \beta_2 x_2^3 - c_2 x_1 &= 0, \end{aligned} \quad (1)$$

where δ_1 and δ_2 are the damping coefficients, ω_1 and ω_2 are the natural frequencies, β_1 and β_2 are the nonlinear coefficients, and the coupling between the oscillators is characterized by the coefficients c_1 and c_2 . The first oscillator (x_1) is submitted to the action of an external harmonic force with amplitude P_0 and frequency ω .

As it is generally characteristic of actual nonlinear systems, weak nonlinearity ($\beta_1 \ll 1$, $\beta_2 \ll 1$), weak coupling ($c_1 \ll 1$, $c_2 \ll 1$) and low damping ($\delta_1 \ll 1$, $\delta_2 \ll 1$) can be supposed. Additionally we assume that the driving harmonic force has a small amplitude ($P_0 \ll 1$), and the resonant frequencies and the frequency of the driving force are closely spaced ($\omega_1 \sim \omega_2 \sim \omega$). Under such quasi-linear conditions the method of slowly varying amplitude [34] can be applied to solve the set of nonlinear equations (1). In the framework of this method the solution of the system (1) is sought in the form $x_1(t) = A \cos(\omega t + \theta)$ and $x_2(t) = B \cos(\omega t + \phi)$, where A and B are the slowly varying amplitudes, and θ and ϕ are the slowly varying phases.

Making the standard change of variables [34] and subsequent averaging we arrive to the following system of reduced equations

$$\begin{aligned} \frac{dA}{d\tau} &= -A - k_1 B \sin(\theta - \phi) - P \sin \theta, \\ A \frac{d\theta}{d\tau} &= -\Omega A + \gamma_1 A^3 - k_1 B \cos(\theta - \phi) - P \cos \theta, \\ \frac{dB}{d\tau} &= -\delta B - k_2 A \sin(\phi - \theta), \\ B \frac{d\phi}{d\tau} &= -(\Omega - \eta) B + \gamma_2 B^3 - k_2 A \cos(\phi - \theta), \end{aligned} \quad (2)$$

where $\tau = \delta_1 t$ is the “slow” time, $\gamma_1 = 3\omega\beta_1/8\delta_1$ and $\gamma_2 = 3\omega\beta_2/8\delta_1$ are the normalized nonlinear coefficients, $k_1 = c_1/2\omega\delta_1$ and $k_2 = c_2/2\omega\delta_1$ are the normalized coupling

coefficients, $\delta = \delta_2/\delta_1$ is the relative damping, $\Omega = (\omega^2 - \omega_1^2)/2\omega\delta_1$ is the frequency mismatch, $\eta = (\omega_2^2 - \omega_1^2)/2\omega\delta_1$ is the frequency difference and $P = P_0/2\omega\delta_1$ is the normalized amplitude of the driving force. The set of equations (2) has a steady-state solution as $\tau \rightarrow \infty$. Practically, to find the steady-state solution we use the conditions of constant amplitudes and phases, i.e. we set $dA/d\tau = dB/d\tau = d\theta/d\tau = d\phi/d\tau = 0$. Thus, the steady-state solution of the system (2) satisfies the following system of algebraic equations

$$\begin{aligned} A^2 &= \frac{B^2}{k_2^2} [\delta^2 + (\Omega - \eta - \gamma_2 B^2)^2], \\ \left[A^2 + \frac{k_1}{k_2} \delta B^2 \right]^2 + \left[\Omega A^2 - \gamma_1 A^4 - \frac{k_1}{k_2} B^2 (\Omega - \eta - \gamma_2 B^2) \right]^2 &= A^2 P^2. \end{aligned} \quad (3)$$

It defines the steady-state amplitudes of small nonlinear oscillations existing in the set of two coupled Duffing oscillators (1).

As it is well known [9], if the nonlinearity in the system (1) is absent ($\beta_1 = \beta_2 = 0$) each of the two linear oscillators has two resonant states at some particular frequencies. In such a linear system the amplitudes can be calculated exactly using the complex amplitude method. Assuming notations used here they have the following form

$$A = \left| \frac{(-\Omega + \eta + i\delta) P}{(\Omega - i)(\Omega - \eta - i\delta) - k_1 k_2} \right|, \quad B = \left| \frac{k_2 P}{(\Omega - i)(\Omega - \eta - i\delta) - k_1 k_2} \right|. \quad (4)$$

It should be noted that in order to simplify derivation of the amplitudes (4) we use a complex harmonic exponent instead of a real cosine function in the right hand side of the first equation in (1).

Typical dependences of the amplitudes (4) on the frequency Ω are presented in Fig. 1. The spectral line of the second linear oscillator (which is unforced) has two resonances with symmetrical Lorentzian shape positioned nearly the corresponding eigenfrequencies (see the red dash line in Fig. 1). At once the spectral line of the first oscillator (which is forced) has two resonances with both symmetrical Lorentzian and asymmetrical Fano shape (see the blue solid line in Fig. 1). The Fano resonance is a result of the composition (interference) of two oscillations from the driving force and the second coupled oscillator. If the phase of the oscillator changes monotonously when the driving frequency passes through the resonance the Lorentzian resonance is observed. On the contrary if the phase dependence has a gap then the Fano resonance appears [9]. In particular for the considered system the antiresonant state takes place at the frequency $\Omega = \eta = -5$.

The shape of the linear resonances does not depend on the amplitude of the driving force. In other words, changing the amplitude P in equations (4) only leads to scaling along the ordinate axis in Fig. 1. But it is not the same in the case of nonlinear system ($\beta_1 \neq 0$, $\beta_2 \neq 0$). Indeed, one can see the complicated dependences of the steady-state amplitudes on the driving force in equations (3). Peculiarity of the nonlinear

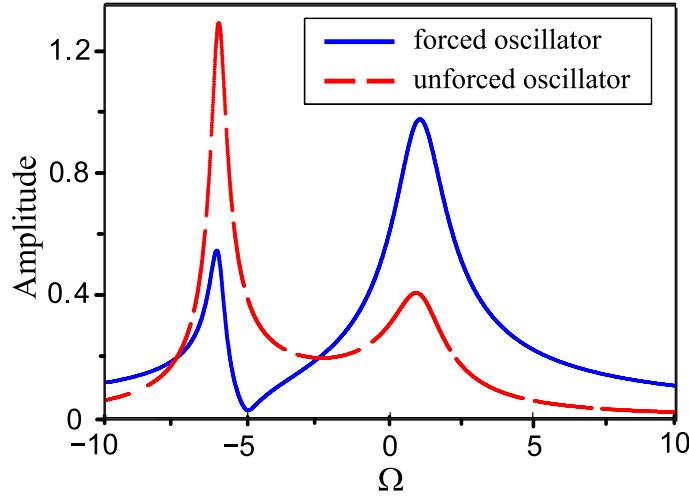


Figure 1. Resonant features of amplitudes of two coupled oscillators in the linear system; $P = 1$, $\delta = 0.15$, $\eta = -5$, $k_1 = k_2 = 2.5$, $\gamma_1 = \gamma_2 = 0$.

resonance consists in the presence of significant dependence of the resonant shapes on the amplitude of the driving force. Such frequency dependences of the steady-state amplitudes (3) on the driving force P are represented in Fig. 2. These curves illustrate the transformations of the coupled Lorentzian and Fano resonances in the system (1) having weak nonlinearity. For the small amplitudes of the driving force ($P = 0.5$) there is no significant difference between the nonlinear resonances features and linear ones. Further increasing the amplitude of the driving force leads to some deformation of the resonant curves. The peaks of Lorentzian resonances become frequency shifted and their shape gets bended. The form of the spectrum line of the left resonant state transforms into a closed loop while the frequency of the antiresonant state acquires a shift [Fig. 2(b),(c)].

It is known that for each nonlinear resonance there is some critical amplitude of the driving force. If the amplitude of the driving force is greater than the critical one then points with a vertical tangent line appear on the resonant curve. It should be noted that the left and the right resonances of the same resonant curve have different critical amplitudes. But the corresponding resonances of the different curves have the same critical amplitudes. When the amplitude of the driving force is high enough the frequency bands appear where the steady-state amplitudes have an ambiguous dependence (hysteresis) on the frequency of the driving force. Some parts of the resonant curves within these bands become unstable sets. Therefore, continuous variation of the driving force frequency leads to jumping of the steady-state amplitudes on the boundaries of the unstable regions which results in appearance of bistability.

For the certain parameters of the system (1) overlapping of the two different resonances can be observed. Evidently, in this case, the spectral curves acquire more than two stable states (which are marked in Fig. 2(c) with circles), i.e., the effect of multistability arises in the system.

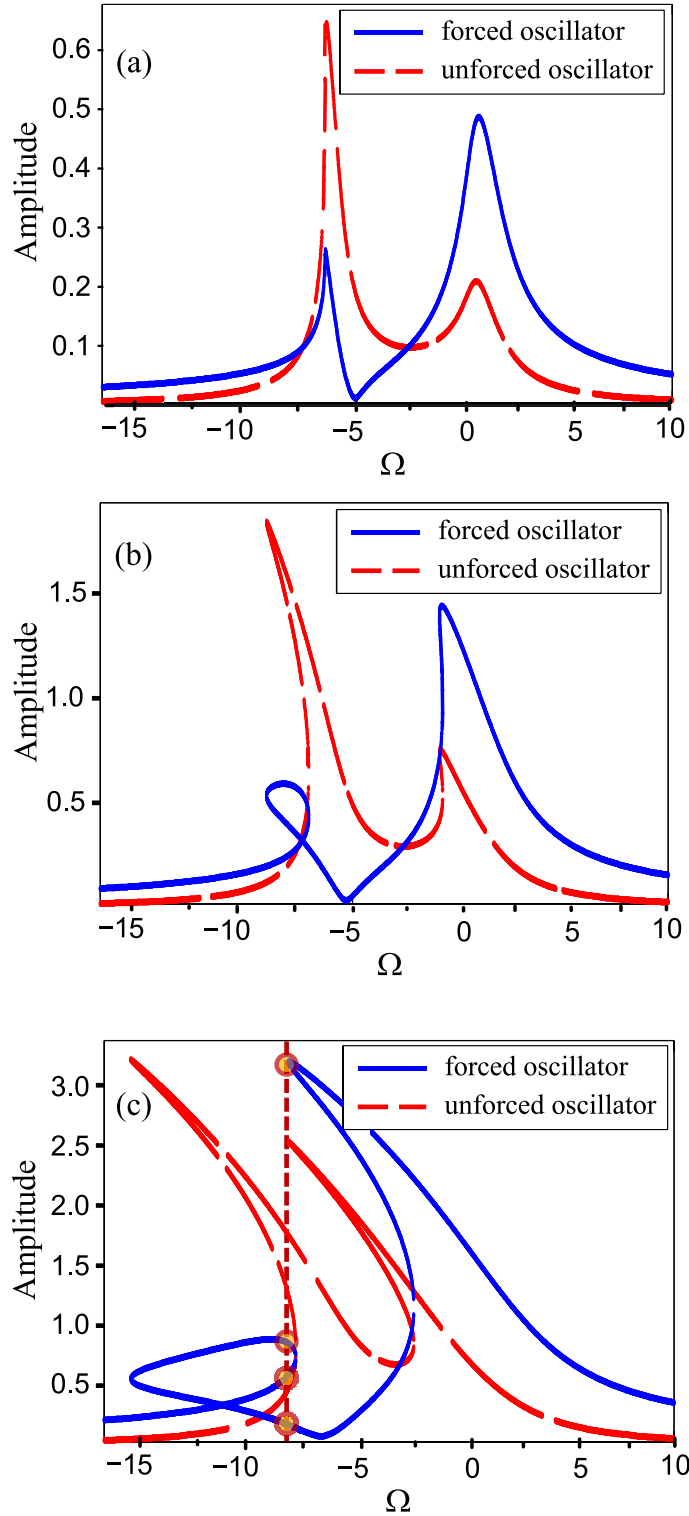


Figure 2. Amplitude-frequency responses of coupled nonlinear oscillators for different amplitudes of the driven force P ; $\delta = 0.15$, $\eta = -5$, $k_1 = k_2 = 2.5$, $\gamma_1 = \gamma_2 = -1$; (a) $P = 0.5$; (b) $P = 1.5$; (c) $P = 3.5$.

3. Electromagnetic analog: Nonlinear bilayer fish-scale metamaterial

Subsequently, in this section, we confirm the predictions of our nonlinear two-oscillator model in an optical system. As such a system we consider a particular configuration of a metamaterial in which resonant-antiresonant states can be excited effectively using trapped-modes. It consists of equidistant arrays of continuous meander metallic strips placed on both sides of a thin dielectric substrate (bilayer fish-scale structures [33]). In such a fish-scale structure the trapped mode resonances can be excited if the incident field is polarized along the strips and when the form of these strips is slightly different from the straight line. Furthermore, in the bilayer structure, besides the trapped-mode resonance excited within each grating, another trapped mode resonance can appear due to a specific interaction of the antiphase current oscillations between two adjacent gratings. Thereby our structure supports two distinct resonant states which corresponds to the characteristic of the two-oscillator model.

A sketch of the studied structure is presented in Fig. 3. The structure consists of two gratings of planar perfectly conducting infinite wavy-line strips placed on both sides of a dielectric slab with thickness h and permittivity ε . The elementary translation cell of the structure under study is a square with sides $d = d_x = d_y$. The full length of the strip within the elementary translation cell is S . Suppose that the thickness h and size d are less than the wavelength λ of the incident electromagnetic radiation ($h \ll \lambda$, $d < \lambda$). The width of the metal strips and their deviation from the straight line are $2w$ and Δ , respectively.

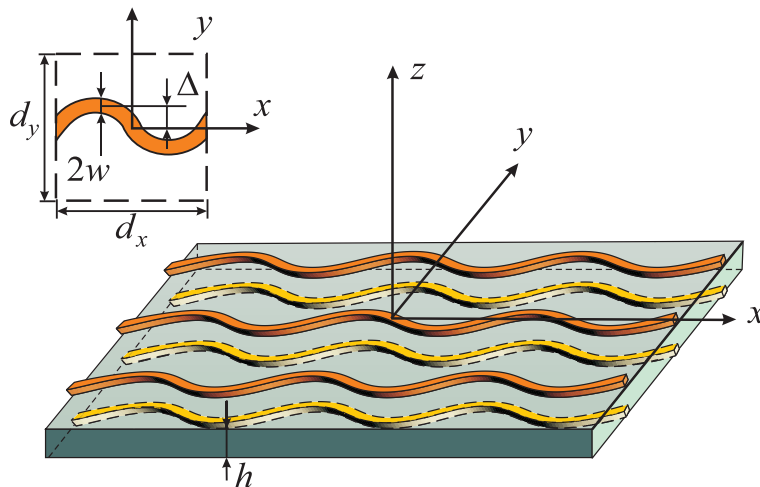


Figure 3. Fragment of a planar bilayer fish-scale metamaterial and its unit cell.

Assume that the normally incident field is a plane monochromatic wave polarized parallel to the strips (x -polarization), and the amplitude of the primary field is A_0 .

In the frequency domain we use the method of moments to solve the problem of electromagnetic wave scattering by the bilayer fish-scale metamaterial [35, 36]. It involves solving the integral equation related to the surface currents which are induced in the metal pattern by the field of the incident wave. In the framework of the method

of moments, the metal pattern is treated as a perfect conductor, while the substrate is assumed to be a lossy dielectric. In the bilayer configuration the method of solution rigorously takes into account an electromagnetic coupling between two adjacent gratings via evanescent partial spatial waves. The metamaterial response can be expressed through the induced currents J_1 and J_2 which flow along the strips of the corresponding grating and the reflection R and transmission T coefficients as functions of normalized frequency ($\alpha = d/l$), permittivity (ε) and other parameters of the structure.

Remarkably, due to the bilayer configuration of the structure under study, there are two possible current distributions which cause the trapped mode resonances. The first distribution is the antiphase current oscillations in arcs of each grating. The currents flow in the same manner on both gratings and the resonance exists due to the curvilinear form of the strips. This resonance is inherent to both single-layer and bilayer structure's configurations [35, 36]. The resonant frequency is labeled in Fig. 4(a) by the letter α_1 , and it corresponds to the first resonant frequency of our two-oscillator model. The second distribution is the antiphase current oscillations excited between two adjacent gratings. Obviously this resonance can be excited only in the bilayer structure's configuration. The resonant frequency is labeled in Fig. 4(b) by the letter α_2 , and hence it corresponds to the second resonant frequency of the two-oscillator model.

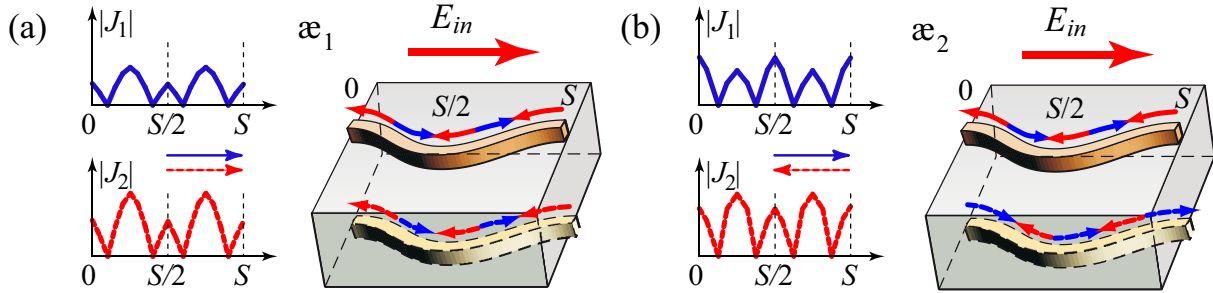


Figure 4. The surface current distribution along the strips placed on the upper and bottom sides of the substrate in the bilayer fish-scale metamaterial.

Thereby the current oscillations on the upper and bottom gratings are characterized with two resonant states which amplitudes are presented in Fig. 5. In contrast to the spectral line of the current amplitude related to the bottom grating, the corresponding spectral line related to the upper grating has a specific asymmetric form with antiresonant state that is in full compliance with predictions of the two-oscillator model. However a small dip in the amplitude of the Lorentzian resonance related to the bottom grating is explained by its incomplete screening with the upper grating.

For a particular bilayer fish-scale structure, two resonant states correspond to two peaks of reflectivity while the antiresonant state corresponds to the maximum of transmissivity. At once these two resonant states have different quality factors. The quality factor of the first resonance depends on the form of strips and is practically independent of the substrate permittivity ε . Thus the less the form of strips is different from the straight line, the greater is the quality factor of the first trapped-mode

resonance. On the other hand, the quality factor of the second resonance crucially depends on both the distance between gratings and permittivity of the substrate. Thus varying the distance between gratings and substrate permittivity changes the trapped-mode resonant conditions and this changing manifests itself in the current amplitudes J_1 and J_2 . We argue that, due to such current distributions the field turns out to be localized between the gratings, i.e. directly in the substrate, which can sufficiently enhance the nonlinear effects if the substrate is made of a field intensity dependent (nonlinear) material.

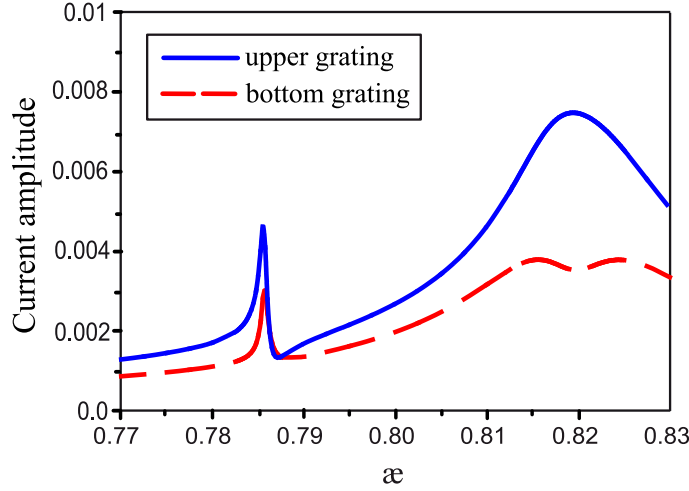


Figure 5. The frequency dependences ($\æ = d/\lambda$) of current amplitudes induced on the upper and bottom gratings; $\varepsilon = 3$, $2w/d = 0.05$, $h/d = 0.2$, $\Delta/d = 0.25$.

In this case, permittivity of the substrate ε becomes to be dependent on the intensity of the electromagnetic field inside it ($\varepsilon = \varepsilon_1 + \varepsilon_2 |E_{in}|^2$). In [22, 23, 24, 37] an approximate treatment was proposed to solve such a nonlinear problem. It is obtained by introducing two approximations. The first one postulates that the inner field intensity is directly proportional to the square of the current amplitude averaged over a metal pattern extent, $I_{in} \sim \bar{J}^2$, where $J = (J_1 + J_2)/2$. The second approximation assumes that, in view of the smallness of the elementary translation cell of the array ($d < \lambda$), the nonlinear substrate remains to be a homogeneous dielectric slab under an action of the intensive light. These approximations allow us obtaining a nonlinear equation related to the current amplitude averaged over metal pattern extent within an elementary translation cell. The input field amplitude A_0 is a parameter of this nonlinear equation. So, at a fixed frequency $\æ$, the solution of this equation gives us the averaged current amplitude \bar{J} which depends on the amplitude of the incident field A_0 . On the basis of the current $\bar{J}(A_0)$ found by a numerical solution of the nonlinear equation, the actual value of permittivity ε of the nonlinear substrate is determined and the reflection R and transmission T coefficients can be calculated as functions of the frequency $\æ$ and the amplitude of the incident field A_0 . For further details about the method of solution the reader is referred to [37].

One can see that as the amplitude of the incident field rises, the frequency

dependences of the inner field intensity acquire a form of bent resonances (Fig. 6) which completely confirm the assumption of our nonlinear two-oscillator model. As mentioned above, such form of lines is a result of the nonlinearly-induced shift of the resonant frequency. In particular, in the optical system, when the frequency of the incident wave is tuned nearly the resonant frequency, the field localization produces growing the inner light intensity which can alter the permittivity enough to shift the resonant frequency [37]. When this shift brings the excitation closer to the resonant condition, even more field is localized in the system, which further enhances the shift of resonance. This positive feedback leads to formation of the hysteresis loop in the inner field intensity with respect to the incident field amplitude, and, as a result, under a certain amplitude of the incident field, the frequency dependences of the inner field intensity take a form of bent resonances. Evidently, in the certain frequency bands, the transmission coefficient acquires two stable states where the effect of bistability takes place.

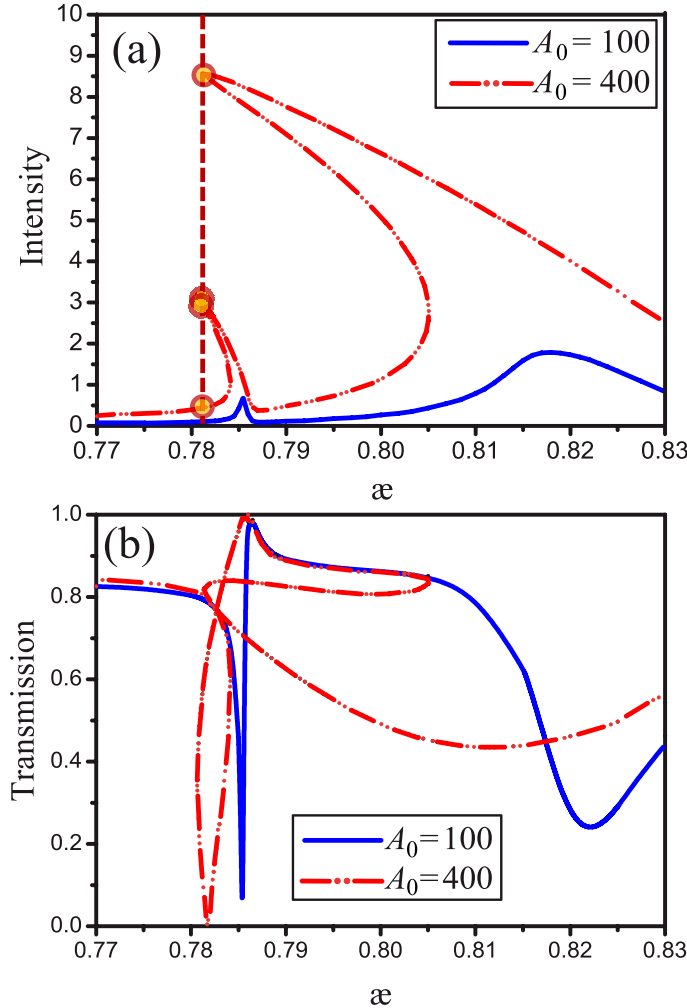


Figure 6. The frequency dependences ($\bar{\omega} = d/\lambda$) of (a) the inner field intensity and (b) the transmission coefficient amplitude; $\varepsilon_1 = 3$, $\varepsilon_2 = 0.005 \text{ cm}^2 \text{ kW}^{-1}$. Other parameters are the same as in Fig. 5.

An important point is that in our system this bending is different for distinctive resonances due to difference in their nature and, respectively, in their current amplitudes [37]. It results in a specific distortion of the curves of the transmission coefficient amplitude nearly the trapped-mode resonant frequencies. Thus, at the frequency $\omega_1 \sim 0.78$ the inner field produced by antiphase current oscillations is confined in the area in the vicinity of each grating and it weakly affects on the permittivity of dielectric substrate. In this case the resonant line acquires a transformation into a closed loop which is a dedicated characteristic of sharp nonlinear Fano-shaped resonances and is related to the characteristic of our nonlinear two-oscillator model. The second resonance $\omega_2 \sim 0.82$ is smooth but the current oscillations produce the strong field concentration between two adjacent gratings directly inside the dielectric substrate. It leads to a considerable distortion of the transmission coefficient amplitude in a wide frequency range, and at a certain incident field amplitude this resonance reaches the first one and tends to overlap it [Fig. 6(b)]. Thus, in this case, the transmission coefficient acquires more than two stable states, i.e. the effect of multistability arises in full accordance to the prediction of the two-oscillator model.

4. Conclusions

In the present paper a direct analogy in oscillation characteristics of two nonlinear systems is evolved. As such systems a chain of coupled Duffing oscillators and an optical structure in the form of bilayer fish-scale metamaterial bearing trapped modes are considered. It is shown that the spectral features of both systems are distinguished by two resonant and single antiresonant states which profiles acquire Lorentzian and Fano forms, correspondingly.

Certain peculiarities of nonlinear impact on spectral lines changing with rising the amplitude of the driving force and the intensity of the incident field are studied for the two-oscillator model and the planar metamaterial, respectively. In the nonlinear regime, resonance bending, closed loop formation, effects of bistability and multistability in the spectra of both structures have been demonstrated.

We argue that our nonlinear two-oscillator model can be used to reveal physical nature of the resonant behavior of such a complicate optical system and can help to identify conditions for the appearance in it of chaotic oscillations, synchronization phenomenon, and another related nonlinear effects.

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